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UTILIZATION OF MODEL PASSIVE IMPURITY CONCENTRATION DISTRIBUTION FUNCTIONS TO COMPUTE TURBULENT FLOW RADIATION

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The adequacy of different semi-empirical methods of taking account of passive impurity concentration fluctuations is investigated for numerical modelling of radiating turbulent flows on the basis of comparisons between computed and measured energetic brightness fields.

The rise in the accuracy of computations of energetic brightness fields of turbulent heated gas flows is associated with the solution of the problem of the influence of temperature and concentration fluctuations on the optical characteristics of a medium. The contribution of turbulent fluctuations to IR radiation of a heated gas jet was investigated in [1]. Since the range of temperature variation in the jet was not large (approximately 300-700 K), the fluctuation characteristics of the temperature field were considered similar to the fluctuation characteristics of the passive impurity concentration field. An analogous approach is used in the present research also. It was shown in [1] that satisfactory agreement between the experimental and computed data is achieved when using probability density functions (PDF) in which the appearance of intermittency in the jet is taken into account in a model fashion. In recent years, intermittency in jet type flows has been investigated quite intensively both theoretically and experimentally [2]. A number of PDF models has been proposed for passive impurity concentration with the intermittency taken into account [2-5]. The purpose of this paper is to confirm the possibility of utilizing such PDF to compute the radiation. Moreover, the influence of temperature and concentration fluctuations on the radiation is studied as a function of the initial turbulence level in the stream.

The measurements and computations were preformed for an axisymmetric subsonic heated jet. A description of the experimental installation and the method of measuring the gas dynamic parameters and the spectrum characteristics are presented in [1, 6].

The jet efflux conditions were changed by using different reducers for an unchanged mode of combustion chamber operation. Three modes were realized: mode 1 without the reducer (jet initial section radius R_0 = 15 cm, initial efflux velocity u_0 = 13 m/sec), mode 2 with two

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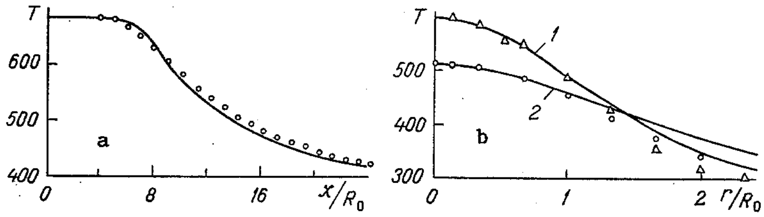

Fig. 1. Comparison of computed (curves) and measured (points) mean values of the temperature: a) axial distributions, mode 3; b) radial distributions, mode 1; 1) $x = 2.9R_0$ and 2) $x = 5.8R_0$, T, K.

axisymmetric reducers (R_0 = 10 cm, u_0 = 23 m/sec), and mode 3 (R_0 = 5 cm, u_0 = 130 m/sec). The initial jet working gas temperature was T_0 = 675 K for all three efflux modes. Jet radiation in the IR band was determined by the presence of CO_2 therein (0.022 atm partial pressure at the nozzle exit), H_2O (0.035 atm), and CO (0.0004 atm). Both N_2 and O_2 are also in the working mixture composition in practically the same relationship as in air.

Computation of the mean jet gasdynamic parameters was performed by using a ε model of turbulent viscosity. Within the framework of this model the turbulent viscosity is determined in terms of two characteristics of the turbulence field, the fluctuation kinetic energy k and the fluctuation energy dissipation rate into heat ε. As is known utilization of reducers results in diminution of the relative level of the fluctuation kinetic energy. The appropriate linear theory governing the dependence of the fluctuation kinetic energy on the degree of compression C (i.e., the ratio between the inlet and exit sections of the reducers) was proposed by Batchelor [7]. The ratio between the fluctuation kinetic energies at the reducer outlet and inlet $k^{(1)}$ and $k^{(0)}$ within the framework of the theory mentioned with a correction obtained on the basis of experimental studies [8] taken into account, is determined by the expression

$$k^{(1)}/k^{(0)} \approx \left(C/2 + \frac{1}{3} C^{-2/3}\right). \qquad (1)$$

Therefore, the initial relative level of turbulent fluctuation kinetic energy k_0/u_0^2 for mode 1 exceeds the analogous ratios for modes 2 and 3 by approximately 4 and 20 times, respectively.

The relationship (1) was used to convert the initial values of k_0 and ε_0 during the passage from one efflux mode to another. Relied upon here for ε_0 is the relationship $\varepsilon_0 \sim k_0^{3/2}/\Lambda$ and it was assumed proportional to the dependence of the turbulence scale Λ on the characteristic scale of the flow R_0. Conformity between the computed and measured axial velocity and/or temperature distributions remained the main criterion for selection of the initial value ε_0. Such a correspondence was achieved only after a substantial correction k_0/u_0^2 and $\varepsilon_0 \cdot R_0/u_0^3$ were utilized in the computation: 0.04 and 0.0037 (mode 1), 0.01 and 0.0007 (mode 2), 0.0022 and 0.00015 (mode 3).

The mean gasdynamic parameter distributions computed on the basis of the k − ε-models, that agree with the appropriate measured quantities with good accuracy, are then used to compute the radiation. Separate results of a comparison between theoretical and experimental data for the modes 1 and 3 are represented in Fig. 1. The results of comparisons for the mode 2 are presented in [1].

The computed and measured values of the radiation were compared in the spectrum range 2200-2300 cm^{-1}, where CO_2 (wing of the band is 4.3 μm) induces the main contribution to jet radiation. Estimates executed in [1] showed that conditions for the applicability of the optically thin fluctuations approximation are satisfied for the jets under investigation in this spectrum range, and can be written in the form

$$\varkappa_\nu \Lambda \ll 1. \qquad (2)$$

The maximal s/d ratios were used here as estimates of the absorption coefficient $\varkappa_\nu$ while the correlation length of the temperature fluctuations was used as the estimate of the characteristic dimension of the fluctuations Λ. Taking account of turbulent fluctuations within

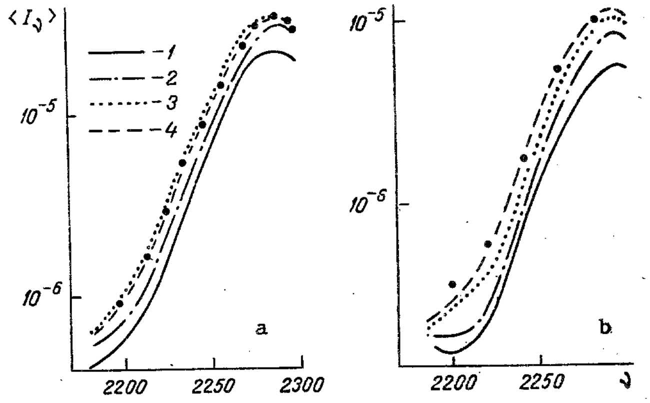

Fig. 2. Comparison of the computed (curves) and measured (points) spectral energetic brightness distributions, mode 2, $x = 4.4R_0$ (a) and $x = 8.8R_0$ (b): 1) radiation computed by the mean thermodynamic parameters; 2, 3, 4) taking account of fluctuations in analysis of the radiation on the basis of the algebraic model, the differential model, and the three-modal approximation, respectively $\langle I_\lambda \rangle$, $W/cm^2 \cdot cp \cdot cm^{-1}$); ν, cm^{-1}.

the framework of the approximation mentioned reduces to local averaging of the optical parameters (absorption coefficients, source functions) in the radiation transport equation. To compute the spectral brightness of the radiation, the following expression [9] will later be used

$$\langle I_\nu \rangle = \int_0^L \left\langle B_\nu \frac{s}{d} \rho \right\rangle \left(1 + \frac{2a^3}{b^2} \frac{\gamma_\nu}{d}\right) \left(1 + \frac{a^2}{b}\right)^{-3/2} \tau dl, \tag{3}$$

where

$$a = \int_l^L \langle s/d \rangle \rho dl'; \; b = 4 \int_l^L \langle s/d \rangle (\gamma_\nu/d) \rho dl';$$

$$\tau = \exp[-a(1 + a^2/b)^{-1/2}].$$

The expression (3) is actually a generalization of a known Curtis–Hudson formula for the case of a turbulent medium in the optically thin fluctuations approximation. In practice the averaging <...> is performed by using model PDF of the temperature and concentration. Since the s/d ratio in the 2200-2300 cm^{-1} range depends quite sharply (exponentially) on the temperature and sufficiently weakly on the concentration, we neglect concentration fluctuations in the radiation computation. The temperature PDF can here be expressed in terms of the concentration PDF of the passive impurity that has the following form in the presence of intermittency in the jet

$$P(z) = \gamma_1 \delta(z - 1) + \gamma_0 \delta(z) + \gamma_z P_t(z). \tag{4}$$

Here γ_1 is the probability of realizing an unmixed working gas of a jet with the concentration $z = 1$, γ_0 is the probability of realizing the unmixed gas of outer space $Z = 0$. The quantity $\gamma^z = 1 - \gamma_1 - \gamma_0$ is often called the coefficient of intermittency. It should be mentioned, however, that the terminology "intermittency coefficient" ordinarily corresponds to the probability γ of realizing a "turbulent fluid", i.e., a fluid in with the vorticity is rot $\mathbf{V} \neq 0$. In the general case, the relationship

$$\gamma_z \leqslant \gamma. \tag{5}$$

is valid for a jet. The equality sign holds for significant distances from the nozzle exit (main section). The inequality $\gamma_Z < \gamma$ [2, 10] denoting vorticity different from zero for unmixed gas volumes with $z = 1$ or 0 can hold in the initial and transition sections. Furthermore, using the terminology "intermittency coefficient" and "turbulent fluid" for the expression (4), we will keep in mind the remark and the relationship (5) presented above.

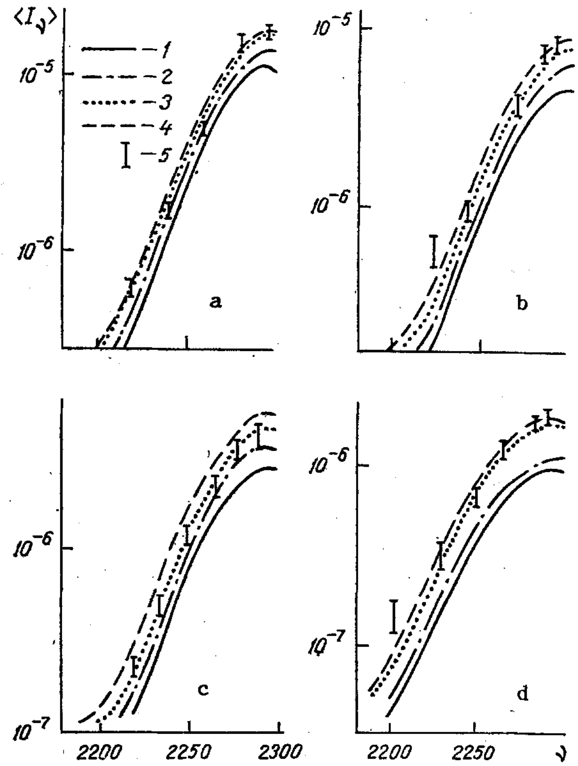

Fig. 3. Comparison of computed and measured spectral energetic brightness distributions, mode 3, $x = 9R_0$ (a), $x = 11.5R_0$ (b), $x = 13R_0$ (c); $x = 17R_0$ (d); notation on the curves the same as in Fig. 2; 5) experimental values.

The following three PDF models of the passive impurity concentration are used in this paper to compute the radiation.

1. ALGEBRAIC MODEL [4] $\langle z\rangle \leqslant 0{,}5$

For

$$\begin{gathered}
\text{a) } \gamma_z = 1,\ \gamma_0 = \gamma_1 = 0 \text{ for } \sigma/\langle z\rangle \leqslant 1/\sqrt{3};\\
\text{b) } \gamma_z = 0.75/(1+\sigma^2/\langle z\rangle^2),\ \gamma_0 = 1-\gamma_z,\ \gamma_1 = 0 \text{ for}\\
1/\sqrt{3} \leqslant \sigma/\langle z\rangle \leqslant \sqrt{2/(3\langle z\rangle)-1};\\
\text{c) } \gamma_z = 6[\langle z\rangle(1-\langle z\rangle)-\sigma^2],\ \gamma_0 = 1-\langle z\rangle-3[\langle z\rangle(1-\langle z\rangle)-\sigma^2]\\
\text{for } \sqrt{2/(3\langle z\rangle)-1} \leqslant \sigma/\langle z\rangle \leqslant \sqrt{1/\langle z\rangle-1}.
\end{gathered} \tag{6}$$

For $\langle z\rangle \geq 0.5$ the relationships a)-c) are used for $\gamma_0 \leftrightarrow \gamma_1$, wherein the substitutions $\langle z\rangle \leftrightarrow 1 - \langle z\rangle$ are realized. The PDF in a turbulent fluid P_t is determined for $\gamma_z < 1$ by the relations

$$\begin{gathered}
P_t(z) = \frac{1}{\langle z\rangle_t} f(z/\langle z\rangle_t),\ f = B_1 A_i(y),\ y = B_2\alpha + B_3,\ \alpha = z/\langle z\rangle_t,\\
B_1 = 1.403;\ B_2 = 1.788;\ B_3 = -2.338,
\end{gathered} \tag{7}$$

$A_i(y)$ are the Airey functions determined by the expression

$$A_i(y) = \frac{1}{\pi}\int_0^\infty \cos(t^3/3 + ty)\,dt.$$

The concentration averaged over the turbulent fluid is determined on the basis of the relationship $\langle z\rangle = \gamma_1 + \gamma_z\langle z\rangle_t$. For $\gamma_z = 1$ the normal distribution is used as PDF.

The advantage of the algebraic model is the simplicity of its realization. Required for the calculation of the PDF are just the local values $\langle z \rangle$ and $\sigma^2 = \langle (z - \langle z \rangle)^2 \rangle$ that can be computed within the framework of many turbulent viscosity models, including the $k - \varepsilon$ models also. However, it is evident that the universal connection between γ_z and γ_1 with $\langle z \rangle$ and σ'independent of the initial jet efflux conditions can hold only for sufficiently large distances from the nozzle exit where the initial conditions are "omitted". This universal connection in the algebraic model is carried over to the non-self-similar jet sections also, which can result in noticeable errors in the modelling of the PDF. The mentioned remark should be kept in mind in computing the radiation since the main part of the radiation emanating from the jet is often determined by the initial and transition sections of the flow.

The algebraic model disadvantages can be eliminated partially by modelling the evolution of the quantities γ_z, γ_1, γ_0 on the basis of differential equations.

2. DIFFERENTIAL MODEL [5]

Within the framework of the differential model γ_z, γ_1 are determined by the equations

$$\rho u \frac{\partial \gamma_z}{\partial x} + \rho v \frac{\partial \gamma_z}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r}\left[\rho r \frac{\nu_t}{\mathrm{Pr}_\gamma}(1-\gamma_z)\frac{\partial \gamma_z}{\partial r}\right) + S_\gamma, \tag{8}$$

$$\rho u \frac{\partial \gamma_1}{\partial x} + \rho v \frac{\partial \gamma_1}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r}\left[\rho r \frac{\nu_t}{\mathrm{Pr}_\gamma}(1-\gamma_z)\frac{\partial \gamma_1}{\partial r}\right) + S_{\gamma 1},$$

$$\mathrm{Pr}_\gamma = 0.7;\ S_\gamma = C_1 \rho (C_2 \gamma_1 + C_3 \gamma_0) \gamma_z (1-\gamma_z) k/\nu_t, \tag{9}$$

$$S_{\gamma 1} = -C_1 C_2 \gamma_1 \gamma_z k/\nu_t,\ C_1 = 2.3;\ C_2 = 0{,}2;\ C_3 = 0.5.$$

Finishing the coefficients in the differential model was realized by using the results of experimental investigations of the intermittency coefficient for the velocity field in the non-self-similar turbulent jet domains [1]. Actually this means that within the framework of the differential model equality of the intermittency coefficients is assumed for the velocity field and the concentration field of the passive impurity. This latter remark should be kept in mind when computing jet radiation for short distances from the nozzle exit. The function $P_t(z)$ is determined in the differential model exactly as in the algebraic model.

3. THREE-MODAL APPROXIMATION [12]

Within the framework of the three-modal approximation $P_t = \delta(z - \langle z \rangle_t)$ is assumed. In this case the following expression holds together with (8)

$$\sigma^2 = \gamma_1 (1 - \langle z \rangle)^2 + \gamma_0 \langle z \rangle^2 + \gamma_z (\langle z \rangle_t - \langle z \rangle)^2. \tag{10}$$

Furthermore, the relationships

$$\gamma_1/\gamma_0 = \gamma_1^*/\gamma_0^*, \tag{11}$$

are postulated for final closure of the model, where γ_1^* and γ_0^* are determined by the differential equations

$$\rho u \frac{\partial \gamma_{1,0}^*}{\partial x} + \rho v \frac{\partial \gamma_{1,0}^*}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r}\left(\rho \frac{\nu_t}{\mathrm{Pr}_\gamma} r \frac{\partial \gamma_{1,0}^*}{\partial r}\right) - \rho \frac{\gamma_{1,0}^*}{\tau^*}, \tag{12}$$

i.e., it is assumed that γ_1^* and γ_0^* vary because of turbulent diffusion exactly the same as $\langle z \rangle$ and, moreover, γ_1^* and γ_0^* dissipate because of molecular mixing with the characteristic time τ^* which is expressed successfully in terms of the characteristic dissipation time σ within the framework of the three-modal approximation.

The three-modal approximation was developed directly for analysis of thermal radiation of nonisothermal turbulent jets, which would determine the nature of the simplification and assumptions to be utilized. The three-modal approximation pretends primarily to simulate the fluctuation field adequately for just small distances from the nozzle exit. The form of the PDF for the far jet field, known to be false (the PDF here is not asymptotically normal), is

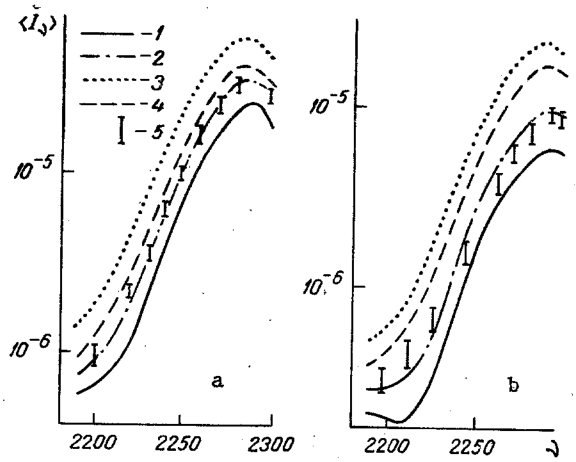

Fig. 4. Comparison of the computed and measured spectral energetic brightness distributions, mode 1, $x = 2.9R_0$ (a), $x = 5.8R_0$ (b); notation of the curves the same as in Fig. 2, 5) experimental values.

justified by the fact that the jet far field radiation for those spectrum ranges where taking account of fluctuation is important is negligibly small. The lack of detailed PDF measurements in the initial and transition sections of the jet did not permit a direct verification of the accuracy of the three-modal approximation. However, it has been shown that this approximation permits taking account of the influence of temperature and concentration fluctuations on the progress of chemical reactions [13] and radiation transport [1] in the non-self-similar sections of a jet in a number of cases.

In this paper, as in [1], the fluctuation contribution to radiation is estimated on the basis of the difference between the measured and computed spectral energetic brightness with respect to the mean thermodynamic parameters. Such an approach is legitimate when the radiation computed by the method utilized and the measured value agree in those jet domains where there are no fluctuations. The mixing layers and their associated turbulent fluctuations induce a negligibly small contribution to the radiation in jet sections abutting on the nozzle exit. The comparisons executed showed that the spectral energetic brightness computed in direct proximity to the nozzle exit by means of the mean thermodynamic parameters agree with the measured value within the limits of experimental error for all three efflux modes.

Results are presented in Figs. 2 and 3 for comparisons for jets obtained by using reducers ("low" initial level of turbulence kinetic energy). It is seen that in the jet near zone the measured radiation can noticeably exceed (2-3 times) the corresponding theoretical values computed by the mean thermodynamic parameters. When analyzing the results we spend the major attention on mode 2 since the gasdynamic and optical parameters of the jet have been measured in greatest detail and most carefully for it. The discrepancy between the computed and measured values of the spectral energetic brightness in the section $x = 4.4R_0$ in mode 2 (Fig. 2a) has been eliminated successfully by using both the differential and the three-modal approximations. The radiation computed within the framework of the algebraic model remains noticeably reduced as compared with the measured values. The reason for the discrepancy is the following in our opinion. For $x = 4.4R_0$ (transition section) the coefficient computed within the framework of the algebraic model is $\gamma_1 = 0$ while the PDF P(z) is Gaussian in the near-axis sections of the jet. These results are a result of carrying over relationships that hold in self-similar flow domains to non-self-similar domains of the jet. According to computations on the basis of (8)-(12), for $x = 4.4R_0$ in near-axial sections $\gamma_1 > 0$ and the appropriate unmixed volumes of heated gas induce a noticeable contribution to the radiation.

The relationships between the results obtained on the basis of using different models remains the same for the section $x = 8.8R_0$. Small but noticeable discrepancies between the results of a computation by the differential model and the experimental data can be associated with the difference discussed earlier, between the quantities γ and γ_z in this part of the jet.

The results obtained for mode 3 (Fig. 3) generally agree with the results of the comparisons for the mode 2. It can be mentioned that the results of computations performed on the basis of the differential model, appear somewhat preferable here as compared with the results of using the three-modal approximation. Even in this case the algebraic model results in noticeably reduced values of the spectral energetic brightness as compared with experiment. The significant spread in the results of measuring the radiation and the lack of detailed measurements of the gasdynamic parameters (only axial distributions were measured for this model) make inexpedient an analysis of the sufficiently fine effects associated with intermittency on the basis of comparing the theoretical and experimental data in this case.

The results obtained for mode 1 (flow without a reducer, "high" initial level of turbulence kinetic energy) are not superposed within the framework of these regularities that were exposed during the analysis of the other two efflux modes (Fig. 4). In this case the discrepancies between the radiation measured and computed by the mean thermodynamic parameters remain noticeable. Satisfactory agreement between the experimental and computed data is successfully obtained here only when utilizing the algebraic model. Let us note that using the algebraic model in computing the radiation in this case is again practically equivalent to the assumption of a normal distribution of the temperature and concentration fluctuations in the near-axis domains of the jet. The differential model and the three-modal approximation result in exaggerated results as compared with the measured values. The greatest discrepancies between the theoretical and experimental data hold when using the differential model. One of the reasons for exaggeration of the computed data might be utilization of the optically thin fluctuations approximation where conditions for its applicability are spoiled [14]. However, both the differential model and the three-modal approximation results in exaggerated results even for the 2200-2300 cm^{-1} spectrum band where the whole jet is optically thin and use of the optically thin fluctuation approximation is know to be justified. The most equally-likely reason for the discrepancies obtained is that for a "high" initial level ofturbulence kinetic volumes of high-temperature working gas of the jet appear weakly in the mixing layer.

NOTATION

x, r, cylindrical coordinates; u,v, longitudinal and transverse mean velocities; ρ, density k, fluctuation kinetic energy; ε, fluctuation kinetic energy dissipation into heat; C, degree of stream narrowing; R_0, jet initial radius; I_ν, energetic brightness spectral density (intensity); $\varkappa_\nu$, spectral absorption coefficient; B_ν, Planck function; L, distance along a ray; Λ, characteristic scale of turbulence; s, integrated force of the spectrum line; $\gamma\nu$, mean line halfwidth; d, mean distance between lines; z, passive impurity concentration; γ, intermittency coefficient for the velocity field; γz, intermittency coefficient for the passive impurity concentration field; γ_1, γ_0, probabilities of realizing unmixed jet working gas and external space gas, respectively σ, variance of the distribution of z; ν_t, kinematic turbulent viscosity; γ_1^* and γ_0^*, quantities being modelled to set up the relationship between γ_1 and γ_0; τ^*, characteristic time of dissipation of γ_1^* and γ_0^*; u_0,k_0, ε_0, values of u, k, ε, in the jet initial section; $P_{r\gamma}$, C_1, C_2, C_3, empirical constants; $\langle ... \rangle$ absolute average with respect to turbulent fluctuations; $\langle ... \rangle_t$, average with respect to the turbulent fluid.

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CALCULATION OF CHARACTERISTICS OF TURBULENT POISEUILLE FLOW

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UDC 532.542.3

A previously described approach is used to calculate decay of a laminar flow into individual turbulized liquid layers. The minimum turbulence scale, turbulent viscosity, and frequency spectrum are determined.

In a previous study [1] the present author proposed a new approach to description of the transition of laminar flow of an incompressible liquid into turbulent flow. That approach is based upon introduction of a distribution function $f(\mathbf{r}'|\mathbf{v}(\mathbf{r}))$, which has the sense of the density of the probability that near the point $\mathbf{r}'$, the liquid will have a transport velocity $\mathbf{v}$ corresponding to the solution of the Navier-Stokes equation for the point $\mathbf{r}$. Thus, the original well-defined hydrodynamic description is complemented by probability relationships which reflect the existence of an intrinsic liquid fluctuation mechanism.

Taking a Gaussian law for the function f, it can be established that the dispersion characteristics are determined by the viscous stress tensor. As is well known (see [2]), the latter defines the production of entropy due to internal dissipative processes. Below we will consider a steady state flow of isothermal incompressible isotropic liquid. In this case for the characteristics referred we have

$$\rho = \frac{2aT}{\eta}\sigma,\ a = \frac{2\beta\eta^2}{R\left(\frac{dp}{dx}\right)^2_{cr}},\ \beta = \text{const.} \tag{1}$$

Turbulization of the laminar Poiseulle flow develops upon satisfaction of two conditions for two adjacent coaxial liquid layers:

$$\rho_1(y_1) - \rho_2(y_2) > \beta(y_2 - y_1), \tag{2}$$

$$\Phi_1(y_1^*,\ \rho_1) = \Phi_2(y_2^*,\ \rho_2),\ y_1^* = y_2^* \in b_2. \tag{3}$$

Here the y-axis is directed from the inner surface of the tube along a radius, the coordinate y^* is determined by the point of intersection of two integral distribution curves on the segment b_2, equal to the thickness of the second layer (the first layer is adjacent to the tube surface ($y_1 < y_2$) and correspondingly $\rho_1 > \rho_2$).

For qualitative estimates condition (3) can be reduced to the simpler expression

$$\rho_1 - \rho_2 \geqslant \frac{\rho_1(b_1 + b_2)}{b_1 + 2b_2}. \tag{4}$$

If we represent the characteristic velocity of a hypothetical laminar flow at a given pressure gradient (head) at one of the points in the layer b_k (which is defined by the condition of conservation of flow, while k is measured from the wall and takes on the values 1, 2, ..., n) as the sum of the two velocities

$$v_k = \bar{v}_k + \delta v_k, \tag{5}$$

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